- Since it is dealing with a single factory that produces all required parts that go into making its final product, all materials necessary for production are simultaneously available at time zero or as needed
- All final products are shipped to the customers immediately
- Transportation time between machines is instantaneous
- A machine can only process one job at a time
- Each job has to be performed on a set of machines in a pre-defined technological ordering
- Processing times are known ahead of time
- Each machine is continuously available for production
- No job splitting is allowed
- There is no job preemption on machines

Model Parameters and Variables

Input Parameters :

P_{ij} = the j^{th} sub-job of final product i. If $j = 0$, then it is the final assembly i.

P_{ijkm} = the k^{th} operation of sub-job j belonging to final product i, where the operation is performed on machine m.

N = number of final product orders to be satisfied ; $i = 1, ..., N$

M = number of machines available for production ; $m = 1, ..., M$

m_{ijk} = the machine required by the k^{th} operation of sub-job j belonging to final product i.

n_i = number of nodes or sub-jobs in the BOM for final product i.

O_{ij} = the set of operations required for processing one unit of P_{ij}.

H_{ij} = number of units of P_{ij} required for meeting the total demand of its direct parent job, if any exists.

$Z(P_{ij})$= the set of parents and grand parents for P_{ij}.

B_i = required batch size of final product i.

Q_i = quantity of P_{ij} required to meet demand.

$$Q_{ij} = B_i * H_{ij}\left(\prod_{P_{ik} \in Z(P_{ij})} H_{ik}\right) \text{ where } \prod_{P_{ik} \in Z(P_{ij})} H_{ik} \geq 1$$

System Variables :

S_{ijkm} = production start time of the k^{th} operation for P_{ij} on machine m.

t_{ijkm} = processing time per unit part required by the k^{th} operation of P_{ij} on machine m.

T_{ij} = total processing time required for making one unit of P_{ij}.

$$\left(T_{ij} = Q_{ij} \sum_{k=1}^{|O_{ij}|} t_{ijkm}\right)$$

F = production flow time (makespan).

α = large positive number

C_{ijkm} = production completion time of the k^{th} operation for P_{ij} on machine m.

Rt_{ijkm} = remaining path time for operations k of P_{ij} on m to realize the final product

$$Rt_{ijkm} = Q_{ij} \sum_{k'=k}^{|O_{ij}|} t_{ijk'm} + \sum_{\forall Z(P_{ij})} T_{ij}$$

$X_{ijkqrsm}$ = 1, if P_{ijk} proceed P_{qrs} on machine m
= 0, otherwise

Model Formulation

The model developed in this study is then formulated as follows:

$$Z_{MILP} = Min\ F \quad (1)$$

$$C_{i(j-1)km} - S_{ij1m'} \leq 0, \ \forall i,j \quad (2)$$

$$C_{ij(k-1)m} - S_{ijkm'} \leq 0, \ \forall i,j,k \quad (3)$$

$$C_{i0Km} - F \leq 0, \ \forall i \quad (4)$$

$$C_{ijkm} - S_{ijkm} = t_{ijkm} * Q_{ij}, \ \forall i,j,k \quad (5)$$

$$C_{qrsm} - C_{ijkm} + \alpha(1 - X_{ijkqrsm}) \geq t_{qrsm} * Q_{qr}, \ \forall i,j,k,q,r,s \quad (6)$$

$$C_{ijkm} - C_{qrsm} + \alpha(X_{ijkqrsm}) \geq t_{ijkm} * Q_{ij}, \ \forall i,j,k,q,r,s \quad (7)$$

$$X_{ijkqrsm} \in \{0, 1\}, \ integer, \ \forall i,j,k \quad (8)$$

$$Q_{ij} \geq 1, \ integer, \ \forall i,j \quad (9)$$